\chapter[Aircraft Noise Prediction]{Medium-Fidelity Aircraft Noise Prediction} \label{C:04\_Noise}

\section{Introduction}\label{S:04\_Introduction}

The proposition of having a surge in air traffic at low altitudes has not come across without justifiable pushback, particularly in terms of noise pollution. From psychoacoustics effects that deal with the cognitive impact of seeing large aircraft to the physically measurable noise, the livelihoods of entire communities run the risk of serious disruption. In the future RUAM ecosystem, noise exposure levels will dictate the approach and departure of air corridors in densely populated areas. It will also be an integral factor in vertiport placement, effectively changing traffic patterns of ground transportation networks.

After dropping the ball on vehicle certification that saw ultimately a large exodus of small UAS and drone companies from US markers in the early 2000s, NASA and the FAA are attempting to move away from their very drawn out, very bureaucratic practices that have come to define these agencies to faster, more streamlined approached that would see an increase in collaboration with academia and the private sector. This includes everything from infrastructure, intelligent systems, air traffic systems, autonomy, vehicle design and performance, business cases, policy creation, flight and ground testing, handling qualities and flight control systems, and of course, safety. In direct response, the application of computational noise prediction tools to complement and potentially inform flight certification processes has become a major area of interest across the private and public sectors. This however means that computational tools must not only be able to predict the noise from isolated rotors but of full aircraft configurations, with multiple rotor blades, performing realistic maneuvers.

Presented below is the implementation of an inexpensive approach capable of accomplishing such a task, enabling fast prediction, iterative design and ultimately. This chapter is broken into four main sections. Section \ref{S:04\_Background} reviews the major sources of aerodynamically generated sounds, as well as the computational approaches to predict acoustic spectra. Following is a brief rev of the A-weighting noise metrics used thought out this work in Section \ref{SS:04\_Noise\_Metrics}. We then dine into the implementation of a model for predicting noise radiated from rotor blades in Section \ref{S:04\_Prediction\_Methods}. The final section of this chapter presents an extensive set of validations tests for the model developed, both against experimental campaigns and higher-fidelity summations.

\section{Background}\label{S:04\_Background}

Before discussing the governing equations of linear acoustics, it is important to first recognize a few characteristics of sound as a wave moving in a body, typically air: 1) Sound takes a finite about of time to travel from the source to the observer, implying that for moving sources such as aircraft, the located at time t when the sound was created is not necessarily where the source would be when it reaches the observer. To correct predict the sound at an observe, one compute the acoustic pressure computed in the retarded timeframe, where retarded time corresponds to the sound emitted at an earlier time $ \tau = t – \frac{R}/{c\_0}$ at a distance $S$ away from an observer; 2) Sound energy can also change due to atmospheric attenuation; 3) Variations in speed can result in changes in wave frequency known as the doppler effect. It is therefore important that these effects are captured when formulating numerical expressions to predict the power spectral density of the source at a given time. Unlike [**Design of quiet efficient propellers**], the assumption that all sources are equidistant from the observer was removed, meaning that the contributions of the acoustic pressure occur at different retarded times.

Excluding the motor, the primary sources of acoustic noise from electric aircraft are harmonic noise, broadband noise and blade vortex interaction or wake noise (1) harmonic noise occurring at integer multiples of the blade passing frequency. The blade passing frequency is computed as the inverse of the product of the number of blades B and the rotational speed N,

Hamonic noise consists of thickness noise that arises from the displacement of the air by the volume of a passing blade element and loading noise due to a combination of lift and drag components on tew rotating blade. As noted by **Aeroacoustics of Flight Vehicles Theory and Practice** Thickness noise can be represented by a monopole source distribution and becomes important at high speeds while loading can be represented as a dipole source and is an important mechanism at low to moderate speeds. If blade movement is confined to the disc plane, the amplitude of two sources varies sinusoidally as the blade rotates around its axis. These two sources of broadband noise are due to turbulent flow impinging on the rotating blades as well as blade self-generated turbulence interacting with the blade trailing edge. Mathematical formulations for both the steady harmonic or tonal sources which dominate the noise spectrum and the broadband component are presented below. Last but not least, blade vortex interaction noise, -- created by unsteady sources that may be time-dependent in the rotating-blade frame of reference, for example, when rotors blades pass through the wake emanating from another blade;

Governing Acoustics Equation

The mathematical formulations for computing the various sources of noise presented in subsequent sections start with the governing equation by [ref 9 from m Aeroacoustics of Flight Vehicles: Theory and Practice. Volume 2: Noise Control ] for the noise radiated from a body in an arbitrary motion

**FWG from paper.**

As noted by [Aeroacoustics of Flight Vehicles: Theory and Practice. Volume 2: Noise Control], it is convenient to consider the blade as the moving body so that the reference frame is rotating with the blade. The term on the left-hand side of the equation represents the acoustic pressure where $p = \rho c\_0 $ while the three terms on the right-hand side, correspond to the thickness, loading and unsteady random noise terms respectively. Examining these three terms a little closer, the first term is simply a time derivative of the summation of thickness source terms while the second is a spatial derivative of the summation of force source terms, both taken at the correct retarded times. The factor $\left| 1 – M\_r \right|$ in the denominator is the Doppler factor

There are two commonly used methods for computing noise from aerodynamic loads, pressure distributions on the surfaces of the vehicle and turbulent wake: Time-domain methods and frequency-domain methods. Time-domain methods require large time histories of the pressures from the loading on each blade to compute radiated noise ﻿Time-domain methods are used to solve the governing equation above directly in terms of the space-time variables. These methods are appealing because they can treat blade geometry with any desired level of precision. The result is the prediction of the acoustic pressure waveform p(t). [Aeroacoustics of Flight Vehicles: Theory and Practice. Volume 2: Noise Control] Some of the most notable implementations include PSU WOPWOP, [ref] ANOPT2, [ref] and ASSPIN (Advanced Subsonic and Supersonic Propeller Induced Noise). On the other hand, frequency-domain approaches eliminate time from the wave equation by means of Fourier transformation but have been known to suffer from loss of information as a result of the oversimplification of the blade geometry. This loss however is less significant for capturing higher harmonics, as pointed out by Hubbard \cite{Hubbard1991a}. Harmonics are computed one at a time and waveforms are generated by summing a Fourier series. Despite time domain methods have demonstrated higher accuracy, they are a few orders of magnitude more expensive in terms of wall clock computing time and memory than frequency-domain methods. A frequency-domain approach was therefore preferred for the scope of this study.

\section{Noise Prediction From A Frequency Domain Approach}\label{S:04\_Prediction\_Methods}

\subsection{Harmonic Noise}\label{SS:04\_Harmonic\_Noise}

Hanson's formation \cite{hanson1995sound} of harmonic propeller noise in the frequency domain was implemented. Extending from early work by \cite{deming1937noise,gutin1948sound,garrick1953theoretical,arnoldi1956propeller,barry1971noise}, this approach encompasses the effects of airfoil thickness, non-axial propeller inflow and blade sweep. The acoustic prediction of the aircraft is confined to rotating blades and excludes fluctuating loads on wings and noise generated in the quasi-steady wake. Go garner appreciation for the method used in this work, we start the derivation from a slightly modified version of the governing equation by Goldstein [ref]

[reference source]

Starting with Goldstein's version of the acoustics analogy which is described as the fundamental equation governing the generation of sound in the presence of sound boundaries

\begin{equation}

\rho^{\prime}(x, t)=-\frac{1}{c\_{0}^{2}} \int\_{-T}^{T} \int\_{S(\tau)}\left(\rho\_{0} V\_{n} \frac{\partial G}{\partial \tau}+f\_{i} \frac{\partial G}{\partial y\_{i}}\right) \mathrm{d} S(y) \mathrm{d} \tau +\frac{1}{c\_{0}^{2}} \int\_{-T}^{T} \int\_{\nu(\tau)} T\_{i j} \frac{\partial^{2} G}{\partial y\_{i} \partial y\_{j}} \mathrm{~d} y \mathrm{~d} \tau \label{eq:04\_goldstein}

\end{equation}

Here, $V\_n$ is the normal surface velocity which will be further alluded to as the monopole source term. The dipole source term in the expressive above is $f\_i$, which represents the $i^{th}$ component of the force per unit area exerted by the fluid on the boundary. This is taken as the dipole source term in the above expression. $T\_{ij}$ is Lighthill's stress tensor [reference] which is used to describe quadrupole sources given as

\begin{equation}

T\_{i j}=\rho u\_{i} u\_{j}+\left(p-c\_{0}^{2} \rho^{\prime}\right) \delta\_{i j} \label{eq:04\_Tij}

\end{equation}

where $u\_i$ is the $i^{th}$ component of the disturbance velocity in the fluid.

The integrals over the source time, $tau$ are over the range $+-T$ large enough to include all source times of interest. To solve this expression, we will utilized a collection of mathematical operations and approximations, the first of them being the free space from of Green's function which is essentially an integral kernal used to solve ODE's and PDE's. This function is given below as

\begin{equation}

G=\frac{\delta\left(t-\tau-R / c\_{0}\right)}{4 \pi R} \label{eq:04\_Greens\_func}

\end{equation}

where $R = \left| x - y \right|$, and $x$ and $y$ are the observer and source locations measures from a reference point fixed in the fluid.

We now examine the motion of the propeller blade in forward flight. First, looking at one blade segment as shown in Figure [] adapted from [], we can replace the Cartesian coordinates with the space fixed, locally orthogonal helicoidal coordinates $\gamma\_i = (\gamma\_0,\zeta\_0,r\_0)$ defined below

\begin{subequations}

\begin{align}

y\_{1}&=\frac{\Omega r\_{0} \xi\_{0}}{U}-\frac{V \gamma\_{0}}{U} \label{eq:04\_y\_1} \\

y\_{2}&=r\_{0} \cos \left(\frac{V \xi\_{0}}{r\_{0} U}+\frac{\Omega \gamma\_{0}}{U}\right) \label{eq:04\_y\_2}\\

y\_{3}&=-r\_{0} \sin \left(\frac{V \xi\_{0}}{r\_{0} U}+\frac{\Omega \gamma\_{0}}{U}\right) \label{eq:04\_y\_3}

\end{align}

\end{subequations}

The first term on the right hand side of Equation [c] captures the distance the blade, travelling at a particular speed, U given in Equation below [ ]has gone in time t while the second term captures the angular offset of the blade from t = 0.

\begin{equation}

U=\sqrt{V^2 + (\Omega R) } \label{eq:04\_U}

\end{equation}

From an observer at (x,y,z), the distance R is given by

\begin{subequations}

\begin{align}

R&=\sqrt{\left(x - y\_1 \right)^{2} + \left(y - y\_2 \right)^{2} + \left(z - y\_3 \right)^{2}} \label{eq:04\_R} \\

R&=\sqrt{\left(x-\frac{\Omega r\_{0} \xi\_{0}}{U}+\frac{V \gamma\_{0}}{U}\right)^{2}+y^{2} -2 y r\_{0} \cos \left(\frac{V \xi\_{0}}{r\_{0} U}+\frac{\Omega \gamma\_{0}}{U}\right) + z^2 + 2 z r\_{0} \cos \left(\frac{V \xi\_{0}}{r\_{0} U}+\frac{\Omega \gamma\_{0}}{U}\right) +r\_{0}^{2}} \label{eq:04\_R\_expanded}

\end{align}

\end{subequations}

Using thin airfoil theory, the approximation to permit the surface boundary conditions to be solved on the mean camber line surface rather than the upper and lower surfaces of the blade. The source strengths $V\_n$ and $f\_i$ are determined from actual geometry but their point of action is on the mean helicoidal surface. (a limitation with this formulation).

Continuing from Equation [ ], for the surface sources, the surface area element in Figure [ref] can be re-expressed as a function of the differential span of the blade element, $r$ and the differential element of the mean camber line surface, $\gamma$ as follows:

\begin{equation}

\mathrm{~d}S = \mathrm{~d}r \mathrm{~d}\gamma \label{eq:04\_dS}

\end{equation}

[green figure]

For the quadroupole source $T\_ij$ which acts throughout the volume surrounding the blades, the volume element becomes $dy = \mathrm{~d}\zeta\_0 \mathrm{~d}\gamma\_0 \mathrm{~d}r\_0 $, leading to Equation [] becoming:

\begin{equation}

p(x, t)=-\int\_{-\infty}^{\infty} \int\_{0}^{r} \int\_{-\infty}^{\infty}\left(\rho\_{0} V\_{n} \frac{\partial G}{\partial \tau}+f\_{i} \frac{\partial G}{\partial \gamma\_{i}}\right) \mathrm{d} \gamma\_{0} \mathrm{~d} r\_{0} \mathrm{~d} \tau +\int\_{-\infty}^{\infty} \int\_{0}^{\infty} \int\_{-\infty}^{\infty} \int T\_{i j} \frac{\partial^{2} G}{\partial \gamma\_{i} \partial \gamma\_{j}} \mathrm{~d} \xi\_{0} \mathrm{~d} \gamma\_{0} \mathrm{~d} r\_{0} \mathrm{~d} \tau \label{eq:04\_pressure\_1}

\end{equation}

The second significant mathematical operation is the expression of surface sources as volume sources. First, the monopole source is given as :

\begin{subequations}

\begin{align}

V\_{n} &= U h^{\prime}\left(\gamma\_{0}, r\_{0}\right) \\

V\_{n} &=U \mathrm{~J} \bar{H}^{\prime}\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right) \mathrm{d} \xi\_{0} \label{eq:04\_monopole\_term}

\end{align}

\end{subequations}

Secondly, the dipole source $f\_i$ at $\tau\_0$ can be written as

\begin{subequations}

\begin{align}

f\_{i}\left(\gamma\_{0}, r\_{0}\right)&=\int F\_{i}\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right) \mathrm{d} \xi\_{0} \\

F\_{i}\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right)&=f\_{i}\left(\gamma\_{0}, r\_{0}\right) \delta\left(\mathrm{FA}+\xi\_{0}\right) \label{eq:04\_dipole\_term}

\end{align}

\end{subequations}

Again the action at the mean surface is expressed by:

\begin{equation}

\bar{H}\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right)=h\left(\gamma\_{0}, r\_{0}\right) \delta\left(\mathrm{FA}+\xi\_{0}\right)

\end{equation}

where $\delta(FA + \zeta\_0 )$ is the Dirac delta function which places the surface sources on the $\zeta\_0$ = -FA surface if the blade is bent or offset as shown in Figure [ref]. We will revisit these source terms later once we have completely reformulated the integral for ease of numerical integration.

The acoustic pressure given by the volume-time integral is now

\begin{equation}

p= \iiiint \left[-\rho\_{0} U \bar{H}^{\prime}\left(\gamma\_{0}+U \tau, r\_{0}, \xi\_{0}\right) \frac{\partial G}{\partial \tau} -F\_{i}\left(\gamma\_{0}+U \tau, r\_{0}, \xi\_{0}\right) \frac{\partial G}{\partial \gamma\_{i}}+T\_{i j}\left(\gamma\_{0}+U \tau, r\_{0}, \xi\_{0}\right) \times \frac{\partial^{2} G}{\partial \gamma\_{i} \partial \gamma\_{j}}\right] \mathrm{d} \xi\_{0} \mathrm{~d} \gamma\_{0} \mathrm{~d} r\_{0} \mathrm{~d} \tau \label{eq:04\_pressure\_2}

\end{equation}

The third significant mathematical operation is the representation of the integrand on the right hand side as one function, $g$, where

\begin{equation}

g\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right)=\rho\_{0} U^{2} \bar{H}^{\prime \prime}\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right)+\frac{\partial}{\partial \gamma\_{i}} F\_{i}\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right) +\frac{\partial^{2}}{\partial \gamma\_{i} \partial \gamma\_{j}} T\_{i j}\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right) \label{eq:04\_generalized\_function}

\end{equation}

We can not rewrite the Equation \label{eq:04\_pressure\_2}as

\begin{equation}

p(x, t)=\iiint \int g\left(\gamma\_{0}+U \tau, r\_{0}, \xi\_{0}\right) \frac{\delta\left(t-\tau-R / c\_{0}\right)}{4 \pi R} \mathrm{~d} \xi\_{0} \mathrm{~d} \gamma\_{0} \mathrm{~d} r\_{0} \mathrm{~d} \tau \label{eq:04\_pressure\_3}

\end{equation}

In this equation above, the $\tau$ integration may be performed first because the spatial integration limits no longer depend on $\tau$ (i.e. no $\sfrac{\partial}{\partial \tau}$. Thus

\begin{equation}

p(x, t)=\iiint \frac{I}{4 \pi R} g\left(\gamma\_{0}+U t-\frac{U R}{c\_{0}}, r\_{0}, \xi\_{0}\right) \mathrm{d} \xi\_{0} \mathrm{~d} \gamma\_{0} \mathrm{~d} r\_{0} \label{eq:04\_pressure\_4}

\end{equation}

Procedure for Far Field Calculations

We first begin with the binomial expansion of Equation \ref{eq:04\_R\_expanded} for radiation points near the origin (blade sections) to the field point $r,\theta$. Retaining terms to first order in $\gamma\_0/r$, $\zeta\_0/r$ and $r\_0/r$ yields

\begin{equation}

R \rightarrow r-\frac{\Omega r\_{0}}{U} \xi\_{0} \cos \theta+\frac{V}{U} \gamma\_{o} \cos \theta-r\_{0} \sin \theta \cos \left(\frac{V \xi\_{0}}{U r\_{0}}+\frac{\Omega \gamma\_{0}}{U}\right) \label{eq:04\_farfeild\_r}

\end{equation}

We utilize the forth significant mathematical operation which is the utilization of a Fourier transformation pair to describe oscillatory nature of the source:

\begin{equation}

\psi\left(\frac{\omega}{U}, r\_{0}, \xi\_{0}\right)=\int\_{-\infty}^{\infty} g\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right) \exp \left[i \frac{\omega}{U} \gamma\_{0}\right] \mathrm{d} \gamma\_{0} \label{eq:04\_fourier\_source\_1}

\end{equation}

where

\begin{equation}

g\left(\gamma\_{0}, r\_{0}, \xi\_{0}\right)=\frac{1}{2 \pi U} \int\_{-\infty}^{\infty} \psi\left(\frac{\omega}{U}, r\_{0}, \xi\_{0}\right) \exp \left[-i \frac{\omega}{U} \gamma\_{0}\right] \mathrm{d} \omega \label{eq:04\_fourier\_source\_2}

\end{equation}

Substitution \ref{eq:04\_fourier\_source\_2} into \ref{eq:04\_pressure\_4} yields

\begin{equation}

p(x, t) =\frac{1}{8 \pi^{2} r} \iiint \frac{1}{U} \int \psi\left(\frac{\omega}{U}, r\_{0}, \xi\_{0}\right) \times \exp &\left[-i \frac{\omega}{U}\left(\gamma\_{0}+U t-\frac{U R}{c\_{0}}\right)\right] \mathrm{d} \omega \mathrm{d} \xi\_{0} \mathrm{~d} \gamma\_{0} \mathrm{~d} r\_{0} \label{eq:04\_pressure\_5}

\end{equation}

But this in the form of a Fourier integral of the form

\begin{equation}

p(x, t)=\int P(x, \omega) e^{-i \omega t} \mathrm{~d} \omega \label{eq:04\_fourier\_integral}

\end{equation}

Thus the Fourier transform of the acoustic pressure is

\begin{equation}

P(x, \omega)=\frac{I}{8 \pi^{2} r} \iiint \frac{1}{U} \psi\left(\frac{\omega}{U}, r\_{0}, \xi\_{0}\right) \times \exp \left[-i \frac{\omega}{U}\left(\gamma\_{0}-\frac{U R}{c\_{0}}\right)\right] \mathrm{d} \xi\_{0} \mathrm{~d} \gamma\_{0} \mathrm{~d} r\_{0} \label{eq:04\_pressure\_6}

\end{equation}

Note here that for the amplitude factor $1/R$, we simply use $R \rightarrow r$. Substituting Equation \ref{eq:04\_farfeild\_r} into Equation \ref{eq:04\_pressure\_6}

\begin{equation}

P(x, \omega)=\frac{\exp \left[i \frac{\omega r\_{0}}{c\_{0}}\right]}{4 \pi r} \iint \psi\left(\frac{\omega}{U}, r\_{0}, \xi\_{0}\right) \times \exp \left[-i \frac{\omega}{c\_{0}} \frac{\Omega r\_{0}}{U} \xi\_{0} \cos \theta\right] I \mathrm{~d} \xi\_{0} \mathrm{~d} r\_{0} \label{eq:04\_pressure\_7}

\end{equation}

where

\begin{equation}

I= \frac{1}{2 \pi U} \int\_{-\infty}^{\infty} \exp \left[-i \frac{\omega}{U}\left(1-M\_{x} \cos \theta\right) \gamma\_{0}\right] \times \exp \left[-i \frac{\omega r\_{0}}{c\_{0}} \sin \theta \cos \left(\frac{\Omega \gamma\_{0}}{U}+\frac{V \xi\_{0}}{U r\_{0}}\right)\right] \mathrm{d} \gamma\_{0} \label{eq:04\_I\_1}

\end{equation}

Here, $M\_x = V/c\_0$. Equation can be rewritten as

\begin{equation}

I= \frac{I}{2 \pi U} \int\_{-\infty}^{\infty} \exp \left[-i \frac{\omega}{U}\left(I-M\_{x} \cos \theta\right) \gamma\_{0}\right]

\times \exp \left[i \frac{\omega r\_{0}}{c\_{0}} \sin \theta \cos \left(\frac{\Omega \gamma\_{0}}{U}+\frac{V \xi\_{0}}{U r\_{0}}-\pi\right)\right] \mathrm{d} \gamma\_{0} \label{eq:04\_I\_2}

\end{equation}

The fifth significant mathematical operation is the expansion of the second exponential with the Bessel generating function,

\begin{equation}

\exp [i z \cos x]=\sum\_{n=-\infty}^{\infty} J\_{n}(z) \exp \left[i n\left(x+\frac{\pi}{2}\right)\right]

\end{equation}

which gives, after rearrangement,

\begin{equation}

I= \sum J\_{n}\left(\frac{\omega r\_{0}}{c\_{0}} \sin \theta\right) \exp \left[i n\left(\frac{V \xi\_{0}}{U r\_{0}}+\frac{\pi}{2}\right)\right] \frac{1}{2 \pi U}

\times \int \exp \left\{i\left[\frac{n \Omega}{U}-\frac{\omega}{U}\left(1-M\_{x} \cos \theta\right)\right] \gamma\_{0}\right\} \mathrm{d} \gamma\_{0 }. \label{eq:04\_I\_3}

\end{equation}

The integral can be evaluated with the Dirac Delta function formulation

\begin{equation}

\delta(\alpha)=\frac{I}{2 \pi} \int\_{-\infty}^{\infty} e^{i \omega x} \mathrm{~d} \omega

\end{equation}

which, when applied to \label{eq:04\_I\_3} becomes

\begin{equation}

I= \frac{1}{I-M\_{x} \cos \theta} \sum\_{n=-\infty}^{\infty} J\_{n}\left(\frac{\omega r\_{0}}{c\_{0}} \sin \theta\right) \times \exp \left[i n\left(\frac{V \xi\_{0}}{U r\_{0}}-\frac{\pi}{2}\right)\right] \delta\left(\omega-\frac{n \Omega}{I-M\_{x} \cos \theta}\right) \label{eq:04\_I\_4}

\end{equation}

from which it follows that the noise at radiation angle B is periodic at the Doppler frequency $(\Omega/\pi)/(l -M\_x cos \theta)$. If we write the discrete spectrum as the sum of the harmonics

\begin{equation}

P(x, \omega)=\sum\_{n=-\infty}^{\infty} P\_{n}(x) \delta\left(\omega-\frac{n \Omega}{1-M\_{x} \cos \theta}\right) \label{eq:04\_pressure\_8}

\end{equation}

where $P\_n$ is the $nth$ complex Fourier coefficient of the acoustic pressure, then

\begin{equation}

P\_{n}(x)=\frac{\exp \left[i n\left(\frac{\Omega\_{D} r}{c\_{0}}-\frac{\pi}{2}\right)\right]}{4 \pi r\left(I-M\_{x} \cos \theta\right)} \int\_{0}^{\infty} J\_{n}\left(\frac{n \Omega\_{D} r\_{0}}{c\_{0}} \sin \theta\right) \times \int\_{-\infty}^{\infty} \psi\left(\frac{n \Omega\_{D}}{U}, \xi\_{0}, r\_{0}\right) \times \exp \left[-i \frac{n}{U}\left(\frac{\Omega \Omega\_{D} r\_{0}}{c\_{0}} \cos \theta-\frac{V}{r\_{0}}\right) \xi\_{0}\right] \mathrm{d} \xi\_{0} \mathrm{~d} r\_{0} \label{eq:04\_pressure\_9}

\end{equation}

where $\Omega\_D = \Omega/(1 = M\_x cos\theta$).

This is the general far-field result for any helically convected source as given by $g$ in Equation \ref{eq:04\_generalized\_function} and its transform in Equation \ref{eq:04\_fourier\_source\_1}. For $B$ blades, the $m^{th}$ harmonic of blade passing frequency is found by setting $n = mB$ and multiplying by $B$. Waveforms can be computed from the Fourier series.

Using the a definition of coordinate systems in Figure \ref{fig:noise\_coordinate\_diagram} below, $\theta$ and $\phi$ denote radiation angles of an arbitrary propeller in a freestream at an angle of attack, $\alpha$ relative to the observer. These two angles are defined in Equation \ref{eq:theta\_phi} along with other geometrical expressions required to fully interpret the diagram.

\begin{figure}

\centering

\mbox{\subfigure[Isolated rotor in forward flight. \label{fig:Noise\_Coordinate\_Diagram\_}]

{\includegraphics[width=0.9\linewidth]

{FIGURES/04\_Images/Noise\_Coordinate\_Diagram\_1.png}}}\qquad

\mbox{\subfigure[Azimuth radiation angle.\label{fig:Noise\_Coordinate\_Diagram\_2}]

{\includegraphics[width=.4\linewidth]

{FIGURES/04\_Images/Noise\_Coordinate\_Diagram\_2.png}}}

\caption{Sound emission reference frames.}

\label{fig:noise\_coordinate\_diagram}

\end{figure}

\begin{equation}

\textrm{where } \theta = cos^{-1}\left(\frac{x}{S}\right) \quad, \quad

\phi = tan^{-1}\left(\frac{z}{y}\right)\quad, \quad S = \sqrt{x^2 + y^2 + z^2} \quad , \quad Y = \sqrt{y^2 + z^2} \label{eq:theta\_phi}

\end{equation}

\noindent Applying the required transformation from the visual frame to the retarded frame using Equations \ref{eq:coordinate\_transformation\_1} and \ref{eq:coordinate\_transformation\_2} result in the corrected distance and radiation angles from the point of emission.

\begin{subequations}

\begin{align}

\theta\_{r} &= cos^{-1}\left(cos(\theta)\sqrt{1 - M\_x^2 sin^2\theta} + M\_x sin^2\theta\right)\label{eq:coordinate\_transformation\_1} \\

S\_r &= \frac{Y}{sin\theta\_r} \label{eq:coordinate\_transformation\_2}

\end{align}

\end{subequations}

Finally, modification of the propeller from the inertial frame to the body-fixed frame oriented $\alpha$ relative to the freestream as shown in Figure \ref{fig:noise\_coordinate\_diagram} yields the following:

\begin{subequations}

\begin{align}

\cos \theta\_{r}^{\prime}&=\cos \theta\_{r} \cos \alpha+\sin \theta\_{r} \sin \phi \sin \alpha \label{eq:theta\_r\_p} \\

\cos \phi^{\prime}&=\frac{\sin \theta\_{r}}{\sin \theta\_{r}^{\prime}} \cos \phi \label{eq:phi\_p}

\end{align}

\end{subequations}

With these definitions, the thickness and loading noise components of the root mean squared pressure of the $n$\textsuperscript{th} rotor is be expressed as:

\begin{multline}

P\_{T{m}\_n} = \frac{-\rho a^{2} B \sin \theta\_{r} \exp \left[i m B\left(\frac{\Omega S\_{r}}{a}+\left(\phi^{\prime}-\frac{\pi}{2}\right)\right)\right]}{4 \sqrt{2} \pi(Y / D)\left(1-M\_x \cos \theta\_{r}\right)} \ldots\\

{} \int\_{h u b}^{t i p} M\_{s}^{2}(h / b) \exp \left(i \phi\_{s}\right) J\_{m B} k\_{x}^{2} \Psi\_{V} d r \label{eq:thickness\_noise}

\end{multline}

\begin{multline}

P\_{L{m}\_n} = \frac{i m B M\_{t} \sin \theta\_{r} \exp \left[i m B\left(\frac{\Omega S\_{r}}{a}+\left(\phi^{\prime}-\frac{\pi}{2}\right)\right)\right]}{2 \sqrt{2} \pi Y r\_{t}\left(1-M\_x \cos \theta\_{r}\right)} \ldots\\

{} \int\_{h u b}^{t i p}\left[\frac{\cos \theta\_{r}^{\prime}}{1-M\_x \cos \theta\_{r}} \frac{d T}{d r}-\frac{1}{r^{2} M\_{t} r\_{t}} \frac{d Q}{d r}\right] \exp \left(i \phi\_{s}\right) J\_{m B} \Psi\_{L} d r \label{eq:loading\_noise}

\end{multline}

where

\begin{equation}

J\_{m B}=J\_{m B}\left(\frac{m B r M\_{t} \sin \theta\_{r}^{\prime}}{1-M\_x \cos \theta\_{r}}\right)

\end{equation}

\noindent

$J\_{m B}$ denotes Bessel functions of the first kind of order $m$ and comes through manipulation of the governing Ffowcs Williams-Hawking equation in the time domain. $m$ is the harmonic number and $B$ is the number of blades. $k\_x$, the ratio of the blade passing frequency to the speed of the aircraft or wavenumber for short is given as:

\begin{equation}

k\_{x}=\frac{2 m B b M\_{t}}{M\_{s}\left(1-M\_x \cos \theta\_{r}\right)}

\end{equation}

\noindent

The phase lag due to the sweep of the propeller, $\phi\_s$, is defined as:

\begin{equation}

\phi\_{s}=\frac{2 m B M\_{t}}{M\_{s}\left(1-M\_x \cos \theta\_{r}\right)} \frac{M C A}{D}

\end{equation}

\noindent $\phi\_{s}$ will be of particular interest in future research as blade sweep has a significant impact on the phase of radiated sound and the onset of the critical blade tip Mach number. The effect of chordwise non-compactness is captured through non-dimensional source transforms $\Psi\_V$ and $\Psi\_L$, which will be examined in succeeding studies. Given below in Equation \ref{eq:psi\_v} and \ref{eq:psi\_l}, the first term approximates the chordwise thickness as a parabolic distribution with maximum thickness at unity while the second represents a uniform lift distribution. Though suitable for conceptional design, a reformulation to accommodate more accurate shape functions will enable topology optimization using a frequency-domain acoustic prediction approach.

\begin{equation}

\hspace\*{2.3cm}\Psi\_{V}\left(k\_{x}\right)=\left\{\begin{array}{cc}

2 / 3 & \text { if } k\_{x}=0 \\

\frac{8}{k\_{x}^{2}}\left[\frac{2}{k\_{x}} \sin \left(\frac{k\_{x}}{2}\right)-\cos \left(\frac{k\_{x}}{2}\right)\right] & \text { if } k\_{x} \neq 0

\end{array}\right.\label{eq:psi\_v}

\end{equation}

\begin{equation}

\Psi\_{L}\left(k\_{x}\right)=\left\{\begin{array}{cc}

1 & \text { if } k\_{x}=0 \\

\frac{2}{k\_{x}} \sin \left(\frac{k\_{x}}{2}\right) & \text { if } k\_{x} \neq 0

\end{array}\right. \label{eq:psi\_l}

\end{equation}

\noindent The total unweighted harmonic sound pressure level from a single rotor is determined through decibel arithmetic of the thickness and loading terms as follows:

\begin{equation}

SPL\_{har\_{{n}\_n}} = 20 log \left(\frac{ P\_{T{m}\_n}+ P\_{L{m}\_n} }{P\_0}\right). \label{eq:raw\_noise}

\end{equation}

The A-weighting scaling to account for human perception is then applied. This expression provided in Equation \ref{eq:A\_weight} was developed by first fitting the A-weighting standard to a continuous polynomial. It is then summed to the sound pressure level of the corresponding frequencies to produced a re-scaled noise spectrum.

\begin{subequations}

\begin{align}

R\_{A}(f) &=\frac{12194^{2} f^{4}}{\left(f^{2}+20.6^{2}\right) \sqrt{\left(f^{2}+107.7^{2}\right)\left(f^{2}+737.9^{2}\right)}\left(f^{2}+12194^{2}\right)} \\

A(f) &= 20 \log \_{10}\left(R\_{A}(f)\right)-20 \log \_{10}\left(R\_{A}(1000)\right) \\

SPL\_{har\_{{m}\_n}}(dBA) &= SPL\_{har\_{{m}\_n}} + A\_f(f)

\end{align} \label{eq:A\_weight}

\end{subequations}

\noindent where $f$ denotes the vector of frequencies of the first 20 harmonics, that is, $f = 2 \pi m \Omega B$, $m = 1,2...20$. The sound pressure level of one rotor can now be similarly computed by logistically summing over the A-weighted harmonic spectrum using Equation \ref{eq:spl\_rot\_harmonic} and again over the total number of rotors using Equation \ref{eq:spl\_tot\_harmonic}.

\begin{subequations}

\begin{align}

SPL\_{\text{har}\_{n}} = 10 log\left( \sum\_{i = 1}^{i = m} 10^{\frac{ SPL\_{\text{har}\_{{i}\_n}}}{10}} \right) \label{eq:spl\_rot\_harmonic} \\

SPL\_{\text{har}\_{\text{tot}}}= 10 log\left( \sum\_{j = 1}^{j = n} 10^{\frac{ SPL\_{\text{har}\_{\_j}}}{10}} \right). \label{eq:spl\_tot\_harmonic}

\end{align} \label{eq:spl\_harmonic}

\end{subequations}

\subsection{Broadband Noise}\label{SS:04\_Broadband\_Noise}

Broadband noise is a culmination of noise generating phenomena. Ref [Rotor broadband noise prediction with comparison to model data by Brooks and Burley] including Blade-Wake Interaction (BWI) noise due to a rotor blades' interaction with the turbulent wakes emulating from preceding blades and self-noise which is generated as a result of the development and shedding of turbulence within the rotor blades' boundary layers and blade vortex interaction (BVI) noise produced by the rotor blade’s interaction with the strong tip vortex, a phenomenon of finite wings, resulting in distinguishable jumps in pressure disturbance at blade passing frequencies. These three major categories of sources are shown in Figure {reproduced below} reproduced from the original paper.

\texstcolor{red}{[figure of flow field encounted by rotor ]}

Self-noise itself is a combination of noise-generation phenomena shown in Figure [ref]. These are briefly summarized below:

\begin{itemize}

\item Trailing edge noise is caused by the scattering of the turbulence pressure field on both the upper and lower surfaces of the rotor blade. This component is one of the more significant noise spectra contributions to the overall self-noise generated.

\item Laminar Boundary Layer - Vortex Shedding (LBL-VS) noise generated at conditions conducive to laminar boundary layer flow on the surfaces of the blade. The rarity of such flow conditions on the blade makes this component of self-noise not a significant component of self-noise.

\item Blunt Trailing Edge (BTE) noise is due to vortex shedding from a less-than-sharp trailing edge.

\item Tip noise due to the finite nature of the lifting surface resulting in the formation and shedding of a tip.

\end{itemize}

In Figure [next figure below ], we see the contributions of each of these broadband noise-generating components to the overall broadband noise prediction methodology outlined in [Helicopter Rotor Trailing-Edge Noise] along with a comparison to experimental data. These tests were conducted on a BO-105 model rotor in 1994 in the Geerman Duch Wind Tunnel as part of the [The data employed in this paper is that of an international cooperative rotor noise research study by researchers from the US Army Aeroflightdynamics Directorate (AFDD), NASA Langley, the German-Dutch Wind Tunnel (DNW), and the Aerospace Research Establishments of France (ONERA) and Germany (DLR). The HART43 (Higher-harmonic control Aeroacoustic Rotor Test) program was conducted in 1994 in the German-Dutch Wind Tunnel (DNW). Details of the test program and major results are reported by Splettstoesser, et al44 and Kube et al4]. The rotor blade was 4 m in diameter had a constant chord length of 0.121 m and RPM of 1040 or a tip Mach around 0.6. This is important to note because though this was a scaled model of a helicopter blade, it is the size of most eVTOL lift-rotor blades (Joby = \textred{number} m, Archer = \textred{number} m , Vertical = \textred{number} m

, Wisk = \textred{number} m).

Thus far, the broadband model in SUAVE's Fidelity One acoustic module only comprises a formulation for trailing-edge noise; the formulations for the remaining components are still under development. It must therefore be noted that in vertical transition and descent flight segments where the wake from the preceding rotor impinges on the blade there may be an underprediction of noise due to the absence of BWI broadband component noise. Additioanlly, with EVTOL configuration, possessing rotors distributed about the aircraft, the BVI noise is projected to be a significant contribution to the overall noise spectrum of the aircraft, particularly in transitioning maneuvers is also absent from the analysis. Future work to seek to develop and validate models for these two integral components.

The trailing edge noise model used in this work is theoretically is based on the work by Li and Lee \cite{Li2019PredictionStudy,KevinLi2021PredictionUCD. This model was chosen over empirical models [Burley and Brooks (Ref. 15)] that use experimentally derived wall pressure spectal shapes [ Brooks-Pope-Marcolini (BPM) , Brooks et al. (Ref. 5] and other theoretical models such Schlinker and Amiet Ref. 10), Kim and George (Ref. 11) [ Prediction of Rotorcraft Broadband Trailing-Edge Noise and Parameter Sensitivity Study Sicheng ] } based on using the theory of Curle (Ref. 8). This model is capable of handling zero pressure gradient flows and nonsymmetric and high-loading airfoil flows and has exhibited. The new model provides the most accurate results for flat plates and airfoils [ Empirical Wall-Pressure Spectral Modeling for Zero and Adverse Pressure Gradient Flow]. That other modes[ Goody , Hu , Rozenberg , Rozenberg and Kamruzzaman ]. The process begins with the discretized loads and local flow incidence angles of all sections of the rotating blades obtained from BEVF routine described in Chapter \ref{C:03\_SUAVE}. A viscous panel method code is then used to compute the boundary layer properties. This evaluation of the surface pressure distribution and the magnitude of the displacement and momentum thicknesses at the trailing edge using a combination of potential theory and laminar and turbulent boundary layer models from Thwaites \cite{thwaites1949approximate} and Head \cite{head1981new} respectively is a departure from the simplifications made by Brooks, Pope and Marcolini \cite{Brooks2016AirfoilPrediction} whose model assume symmetric airfoil sections and utilize empirical estimations for boundary layer properties. The computed variables along the surface are rotor blades are then used in Lee’s wall pressure spectrum model. A more detailed methodology is now presented below. We start the mathematical formation by examining a section of the blade shown in Figure \ref{fig:Propeller\_Noise\_Rotor\_Diagram}. The local coordinate system located at the midspan of each blade segment is shown in the figure, with both the x and y axes lying on the rotor disk plane if the blade segment is not pitched with respect to the rotor disk. Additionally, the blade pitch angle, $\alpha\_p$, is the angle between the rotor plane and the local blade chord.\begin{figure}[!h]

\centering

\includegraphics[width=0.6\textwidth]{ ./FIGURES/04\_Images/Propeller\_Noise\_Rotor\_Diagram}

\caption{Local blade section coordinate system.}\label{fig:Propeller\_Noise\_Rotor\_Diagram}

\end{figure}

Starting with Amiet’s model for power spectral density at an observer location with respect to the upper and loser surfaces of the airfoil can be written as:

\begin{equation}

S\_{p p(U / L)}=\left(\frac{\omega}{a}\right)^{2} c^{2} dr\left(\frac{1}{32 \pi^{2}}\right) \frac{N\_{B}}{2 \pi} \int\_{0}^{2 \pi} D\_{\phi}\left|L\right|^{2} l\_{r} \Phi\_{p p} d \phi

\end{equation}

where $U$ and $L$ represent the upper and lower surfaces of an airfoil, respectively and

\begin{subequations}

\begin{align}

\epsilon^{2}&=X^{2}+\beta^{2}\left(Y^{2}+Z^{2}\right) \\

\beta^{2}&=1-M^{2} \\

\frac{\omega\_{d}}{\omega} &= \frac{1}{1+M\_{r}\left(\frac{X}{R\_{s}}\right)} \\

M\_r &= \frac{\Omega r}{a}.

\end{align}

\end{subequations}

\noindent $D$ is the spectrum directivity as defined in the appendix of \cite{KevinLi2021PredictionUCD}, $\epsilon$ is the doppler shifted frequency and $c$ is the chord length of the blade section. This paper also provides a comprehensive review of the coordination transformation from the observer to the blade section surface, including any out of plane flapping motion of the blade thus the authors refer the reader to this reference. Schlinker and Amiet's model \cite{Schlinker1981HelicopterNoise} for the loading term, $L$ is used in the above formulation. It is given as follows:

\begin{multline}

|L|= \frac{1}{\Lambda} \Biggl| e^{i 2 \Lambda} \biggl[ 1-(1+i) E^{\*}\left(2\left(\bar{K}+\bar{\mu} M+\bar{\gamma} \right) \right) + e^{-i 2 \Lambda} \sqrt{\frac{K+\mu M+\gamma}{\mu x \epsilon+\gamma}}(1+i) + \ldots\\

{}E^{\*}\left(2\left(\frac{\bar{\mu} x}{\epsilon} +\bar{\gamma} \right) \right) \biggr] \Biggr|\end{multline}

where

\begin{subequations}

\begin{align}

\Lambda&=\bar{k}\_{x}- \frac{\bar{\mu} x}{\epsilon} +\bar{\mu} M \\

K&= \frac{\omega\_{d}}{U\_{c}} \\

\mu&= \frac{\omega\_{d} M}{ U \beta^{2}} \\

\gamma^2 &= \left( \frac{\mu}{\epsilon}\right)^{2}\left(x^{2}+\beta^{2} z^{2}\right).

\end{align}

\end{subequations}

\noindent Here, $k\_x$ is the chordwise wave number denied by $\omega/U$ and $mu = M k\_x / beta^2$ and $E^\*$ represents the the complex conjugate of a Fresnel integral defined by

\begin{equation}

E(x) \equiv \frac{1}{\sqrt{2 \pi}} \int\_{0}^{x} e^{i t} \frac{d t}{\sqrt{t}}.

\end{equation}

\noindent The bar over the variables indicated normalization by the semichord of the blade section. Lastly, the wall pressure is given by

\begin{equation}

\frac{\Phi\_{pp}(\omega) U\_{e}}{\tau\_{w}^{2} \delta^{\*}}=\frac{\max \left(a,\left(0.25 \beta\_{c}-0.52\right) a\right)\left(\frac{\omega \delta^{\*}}{U\_{e}}\right)^{2}}{\left.\left[4.76\left(\frac{\omega \delta^{\*}}{U\_{e}}\right)^{0.75}+d^{\*}\right]^{e}+\left(8.8 R\_{T}^{-0.57}\right)\left(\frac{\omega \delta^{\*}}{U\_{e}}\right)\right]^{h^{\*}}}. \label{eq:phi\_broadband}

\end{equation}

Table \ref{tab:spectrum\_properties} below documents the parameters used within the Equation \ref{eq:phi\_broadband}.

\begin{table}[!h]

\caption{Parameters for wall pressure spectrum model.} \label{tab:spectrum\_properties}

\centering

\begin{tabular}{cc}

\hline

\textbf{Parameter} & \textbf{Value} \\ \hline \hline

$\beta\_{c}$ & $ \left(\Theta / \tau\_{w}\right)|d p / d x|$ \\

$a$ & $ 2.28 \Delta^{2}\left(6.13 \Delta^{-0.75}+d\right)^{e}[4.2(\Pi /\Delta)+1]$ \\

$\Delta$ & $\delta / \delta^{\*}$ \\

$\Pi$ & $0.8\left(\beta\_{c}+0.5\right)^{3 / 4}$ \\

$e$ & $3.7+1.5 \beta\_{c}$ \\

$d$ & $4.76(1.4 / \Delta)^{0.75}[0.375 e-1]$ \\

$R\_{T}$ & $\left(\delta / U\_{e}\right)\left(v / u\_{\tau}^{2}\right)$ \\

$ h^{\*}$ & $\min (3,(0.139+ \left.\left.3.1043 \beta\_{c}\right)\right)+7$\\

$d^{\*}$ & if $\left(\beta\_{c}<0.5\right)$, = $\max (1.0,1.5 d)$ , otherwise = d \\ \hline

\end{tabular}

\end{table}

Finally, the far-field SPL for one side of the $i^{th}$ section on the $n$th rotor blade is calculated from the acoustic power spectral density $S\_{pp}$ as

\begin{equation}

\mathrm{SPL}\_{\text {bb}\_{i(U,L)}}=10 \log \_{10}\left(\frac{2 \pi S\_{p p(U,L)}}{P\_0^2 } \right).

\end{equation}

The rotor’s total SPL can now be computed through decibel arithmetic

\begin{equation}

\mathrm{SPL}\_{\text {bb}\_n}=10 \log \_{10} \sum\_{i=1}^{n\_{S}}\left(10^{0.1 \mathrm{SPL}\_{\text {bb}\_{i,U}}}+10^{0.1 \mathrm{SPL}\_{\text {bb}\_{i,L}}}\right)

\end{equation}

\noindent where $n\_S$ is the total number of sections. Likewise, the total SPL of a vehicle with multiple rotors is given in Equation \ref{eq:total\_broadband} below:

\begin{equation}

\mathrm{SPL}\_{\mathrm{bb}\_{\text{tot}}}=10 \log \_{10} \sum\_{j=1}^{n}\left(10^{\left.0.1 \mathrm{SPL}\_{\mathrm{bb}, j}\right)}\right). \label{eq:total\_broadband}

\end{equation}

\subsection{Total Noise}\label{SS:04\_Total\_Noise}

The final step of the noise prediction methodology is the summation of the tonal and broadband components around the aircraft through decibel arithmetic.

Observer or microphone locations where the sound is computed are positioned in the inertial frame on the x-y plane beneath the aircraft, confined to the range of the aircraft in the longitudinal direction and maximum lateral distance of +/- 1000 feet in rectangular array as depicted in Figure \ref{fig:Observer\_Locations}. This suggests that as the aircraft gains altitude, the absolute distance from the noise sources on the aircraft each microphone.

\begin{figure}

\includegraphics[width=\textwidth]{ ./FIGURES/04\_Images/Observer\_Locations\_2}

\caption{Source-observer diagram.}\label{fig:Observer\_Locations}

\end{figure}

\section{Rotor Noise Validation}\label{S:04\_Total\_Noise}

words words words

\subsection{Harmonic Noise}\label{SSS:04\_Harmonic\_Noise\_Validation}

The propeller harmonic noise is validated against experimental data obtained from Weir \cite{Weir1987}. This experimental test campaign was geared towards evaluating the performance of NASA Aircraft Noise Prediction Program (ANOPP) Propeller Analysis System (PAS) tool and examined propeller noise for a Piper Lance aircraft for comparison with FAA wind tunnel and flyover data. The comparisons presented in this paper are a subsect of the results reported in the reference. The tests conducted on a single 2-bladed, 2.03-meter diameter Hartzell F8475 D-4 propeller which has a rounded-tip and Clark-Y airfoil sections are selected for validation here. The in-flow microphones were placed at 60$\degree$ and 90$\degree$ from the propeller axis at a nominal distance of 4 meters. Additional information on the environmental and operating conditions can be found in the reference. Figure \ref{fig:Propeller\_Noise\_Validation\_1} documents the three cases at power coefficients of 0.027, 0.047 and 0.042. The top row of plots corresponds to the SPLs of the first 18 harmonics at the microphone positioned at 60 $\degree$ from the propeller axis while the bottom row corresponds to the microphone positioned at 90 $\degree$. With the exception of the in-plane case at $C\_P$= 0.042, reasonably good agreement of the medium-fidelity model implemented in SUAVE to PAS and the experimental data reported in the literature is achieved. The second validation study tonal noise was that of the directivity of the strength of first two harmonics of around the propeller. Static-rotor tests by Hubbard \cite{Hubbard1950SoundConditions} were used to benchmark the performance of the implemented computational routine. In this experiment, the helical tip Mach number was 0.62 with microphone placed at 30 feet from the propeller hub. Additionally, the geometry of the propeller blades and other test conditions can be found in the reference. The polar plots in Figure \ref{fig:Propeller\_Noise\_Validation\_2} show that the the performance of this semi-empirical prediction routine start to break down as the observer moves away from the plane the rotor or conversely approaching the propeller axis. This is expected given the mathematical formations for the thickness and loading noise which, with simplifications, can be reduced to show that the strongest signatures are around at 25 $\degree$ and 60 $\degree$ respectively. Nevertheless, the results for over 77\% of the sound directivity were within reasonable bounds and was this deemed satisfactory for the purposes of this study.

\begin{figure}[h!]

\centering

\mbox{\subfigure[Case 1.1: $C\_p = 0.027$, $\theta\_{mic} = 60 \degree$.]

{\includegraphics[width=\halfFigWidth]

{FIGURES/04\_Images/Propeller\_Noise\_Validation\_1.png}}}\qquad % space

\mbox{\subfigure[Case 1.2: $C\_p = 0.027$, $\theta\_{mic} = 90 \degree$.]

{\includegraphics[width=\halfFigWidth]

{FIGURES/04\_Images/Propeller\_Noise\_Validation\_1.png}}}

\mbox{\subfigure[Case 2.1: $C\_p = 0.047$, $\theta\_{mic} = 60 \degree$.]

{\includegraphics[width=\halfFigWidth]

{FIGURES/04\_Images/Propeller\_Noise\_Validation\_1.png}}}

\mbox{\subfigure[Case 2.2: $C\_p = 0.047$, $\theta\_{mic} = 90 \degree$.]

{\includegraphics[width=\halfFigWidth]

{FIGURES/04\_Images/Propeller\_Noise\_Validation\_1.png}}}

\mbox{\subfigure[Case 3.1: $C\_p = 0.042$, $\theta\_{mic} = 60 \degree$.]

{\includegraphics[width=\halfFigWidth]

{FIGURES/04\_Images/Propeller\_Noise\_Validation\_1.png}}}

\mbox{\subfigure[Case 3.2: $C\_p = 0.042$, $\theta\_{mic} = 90 \degree$.]

{\includegraphics[width=\halfFigWidth]

{FIGURES/04\_Images/Propeller\_Noise\_Validation\_1.png}}}

\caption{Comparison of SUAVE prediction with PAS and data for windtunnel test cases \cite{Weir1987}.}

\label{fig:Propeller\_Noise\_Validation\_1}

\end{figure}

\begin{figure}[h!]

\centering

\mbox{\subfigure[1\textsuperscript{st} Harmonic.]

{\includegraphics[width=\halfFigWidth]

{FIGURES/04\_Images/Propeller\_Noise\_Validation\_2\_H1.png}}

\label{fig:Propeller\_Noise\_Validation\_2\_H2}}\qquad % space

\mbox{\subfigure[2\textsuperscript{nd} Harmonic.]

{\includegraphics[width=\halfFigWidth]

{FIGURES/04\_Images/Propeller\_Noise\_Validation\_2\_H2.png}}

\label{fig:Propeller\_Noise\_Validation\_2\_H2}}

\caption{Tonal noise directivity validation study.}

\label{fig:Propeller\_Noise\_Validation\_2}%

\end{figure}

\subsection{Broadband Noise}\label{SSS:04\_Harmonic\_Noise\_Validation}

The experimental campaign to identify characters of the PSD of small unmanned aircraft systems by Zawodny et. al \cite{Zawodny2016} is used to assess the accuracy of the implemented noise model. These tests were conducted on an APC 11x4.7 SF rotor blade and DJI 9443 rotor blade at Structural Acoustics Loads and Transmission Anechoic chamber facility at NASA Langley Research center. Isolated broadband noise measurements of the APC blade are discussed here for validation. In contrast to the validation study of the harmonic noise model, that of broadband proved moderately successful. Despite being able to capture the mid to high regions of the spectrum, poor performance is seen near the blade passing frequency of the text conditions(120-160 Hz).

\begin{figure}[h!]

\centering

\includegraphics[width= 0.6\textwidth]{ ./FIGURES/04\_Images/Noise\_Broadband\_Validation.png}

\caption{Trailing-edge broadband noise validation study.}\label{fig:Propeller\_Noise\_Validation\_3}

\end{figure}

\subsection{Total Noise}\label{SS:04\_Isolated\_Rotor}

In Figure \ref{fig:Propeller\_Noise\_Validation\_4}, we show a comparison of the measured and computed \sfrac{1}{3} octave spectrum of the tonal noise the APC rotor discussed in the previous section at a distance of 1.905 and angle of 45 $\degree$ from the propeller plane. Additional information about experimental setup for these static tests can be found in \cite{Zawodny2016}. Through the computational model overpredicts broadband noise near the blade passing frequency, we observe that together with the harmonic noise components that dominate in this range, the overall noise spectrum falls within reasonable bounds of the experimental data. Future work will however seek to improve these medium-fidelity methods for prediction power spectrum densities.

\begin{figure}[h!]

\centering

\includegraphics[width= 0.6\textwidth]{ ./FIGURES/04\_Images/Noise\_Rotor\_Validation.png}

\caption{One-third octave acoustic spectra validation at different rotation rates.}\label{fig:Propeller\_Noise\_Validation\_4}

\end{figure}

\subsection{Comparison Between Frequency and Time Domain Methods}\label{SS:04\_Comparison}

High-fidelity simulations of noise generation in a propeller-driven unmanned aerial vehicle

CFD analysis of the aerodynamics and aeroacoustics of the NASA SR2 propeller

Simulations of noise generated by rotor-rotor interactions at static conditions

words words words

\section{Noise Metrics}\label{SS:04\_Noise\_Metrics}

words words words

Aircraft Noise Characteristics and Metrics

Community noise exposure is an integral factor in vertiport placement

Acoustic footprint of EC/VTOL aircraft will dictate approach and departure air-corridors

Current noise prediction methods fall on the extremities of the computational cost-accuracy spectrum

There is a need for a tool for iterative conceptual design

Currently, the Federal Aviation Administration (FAA) is using the Day-Night Av- erage A-weighted Sound Pressure Level (DNL) to quantify aircraft noise induced annoyance in the communities around airports. Here, the 65 DNL contour is used as a criterion to determine qualification for noise insulation programs. DNL is an equivalent continuous A-weighted sound pressure level with an addition

of 10 dB during the night-time (2200-0700) (ANSI S3.23-1980, 1980). The addition of 10 dB during the night-time reflects the fact that people are more sensitive to noise during the night. This is mainly because the background noise level is reduced at night which causes aircraft events to be more noticeable. DNL as a single number measure for predicting the effects of the long-term ex-

posure of environmental noise was widely accepted. However, some of its drawbacks, mainly the penalty factor for night-time events (10 dB) are often questioned. Also it is felt that the effects of sharpness, tonal- ness, roughness and fluctuation strength

are not adequately accounted for by DNL

A weighting schemes are based on equal loudness contours (ISO 226, 1987, 2003) at different pressure levels. A-weighting weighting factors (wi) related to the different frequency bands are given in Table 2.1. These are derived from the 40 phon equal loudness contour. Although A-weighted sound level is universally accepted for community noise measurement, Fidell, Pearsons, Tabachnick, and Howe (2000b) mentioned that it is inadequate in assessing aircraft noise impact on a community. This inadequacy is because it de- emphasizes low-frequencies below 400 Hz and high-frequencies above 4000 Hz. If a sound component is around 40 phon then A-weighted sound pressure level may be appropriate.

\label{fig:A\_weighting}

\textcolor{red}{A Weigting PLot}

There exist several assessment methods to quantify aircraft noise annoyance in communities around airports. A few noteworthy ones are weighted sound pressure level based ratings, computed loudness and annoyance based ratings, noise level and event-based ratings and statistical centennial based rating. The rationale behind the development of several noise ratings stemmed from the purpose of using a different noise rating that adequately predicts human response to noise \cite{schultz1982community}. Different occasions led to a slightly different rating in each case. More \cite{more2010aircraft} outlines some of their respective advantages and disadvantages. For the purposes of this study, the A-weighted Maximum Noise Level, $SPL\_{Amax}$, measured in dBA is used. This is an instantaneous peak noise level measured at an observer location during the time period in consideration.

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posure of environmental noise was widely accepted. However, some of its drawbacks, mainly the penalty factor for night-time events (10 dB) are often questioned. Also it is felt that the effects of sharpness, tonal- ness, roughness and fluctuation strength

 are not adequately accounted for by DNL

A weighting schemes are based on equal loudness contours (ISO 226, 1987, 2003) at different pressure levels. A-weighting weighting factors (wi) related to the different frequency bands are given in Table 2.1. These are derived from the 40 phon equal loudness contour. Although A-weighted sound level is universally accepted for community noise measurement, Fidell, Pearsons, Tabachnick, and Howe (2000b) mentioned that it is inadequate in assessing aircraft noise impact on a community. This inadequacy is because it de- emphasizes low-frequencies below 400 Hz and high-frequencies above 4000 Hz. If a sound component is around 40 phon then A-weighted sound pressure level may be appropriate.

\label{fig:A\_weighting}

\textcolor{red}{A Weigting PLot}

﻿As discussed in Section II B 2, human ears have different responses at different frequencies. Various

decibel weighting schemes have been proposed to account for this, the most widely used of which is the A-weighting scheme. This scheme applies a response function to a given sound pressure level, in order to compensate for the frequency response of the human ear. The A-weighting response function A(f) as a function of frequency is plotted in Figure 4. Figure 4 reveals that A(f) is maximized at a frequency of approximately 3 kHz, indicating that humans

are particularly sensitive to sounds at this frequency. In order to reduce subjective annoyance, the designer should strive to avoid sound frequencies near 3 kHz as much as possible.

\section{Code Implementation} \label{S:04\_Code\_Implementation}

The computational approach to predict aircraft noise in SUAVE is a post-processing technique and is not a part of the iterative mission-solving routine. That is to say, the radiated sound pressure level is not repeatedly computed at each control point but one time after the kinematics of each flight segment are fully resolved.

\section{Chapter Summary}\label{S:04\_Chapter\_Summary}

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